The Hermite-Hadamard Inequality: Challenges and Perspectives

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Abstract

In this talk, we present some methodological aspects of the Hermite-Hadamard Inequality, its main lines of development, and the possibility of developing and obtaining results in this dynamic area of mathematical sciences.

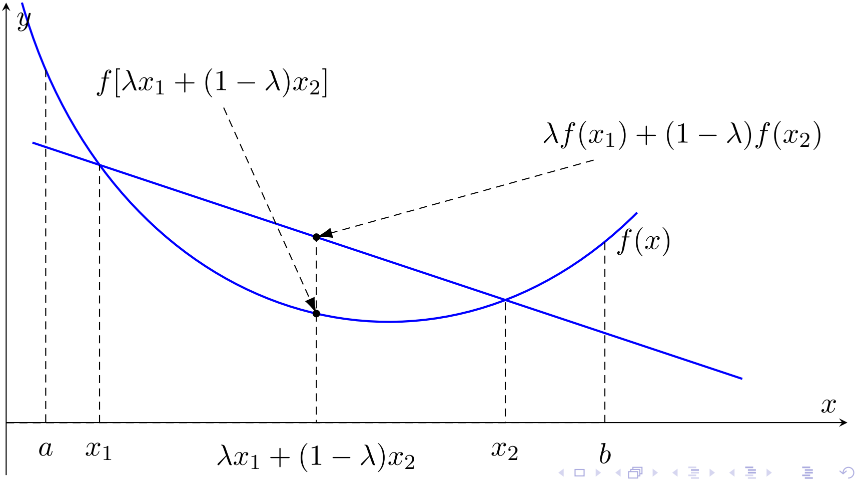
Convex functions play, in contemporary Mathematics, a very prominent role, first of all, because they are especially easy to minimize (for example, any minimum of a convex function is a global minimum).

A function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex on the interval I, if the inequality

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) \tag{0.1}$$

holds for all $x, y \in I$ and $t \in [0,1]$. We say that f is concave if $-f$ is convex.

This notion is attributed to the Danish mathematician Johan Jensen (1859-1925), who unified in a functional class, those functions that verify certain properties.



The Hermite-Hadamard Inequality, central to this talk, is presented in this way.

The inequality ([47])

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{0.2}$$

is true for any convex function f en $[a,b]$.

Convex Functions

[59]

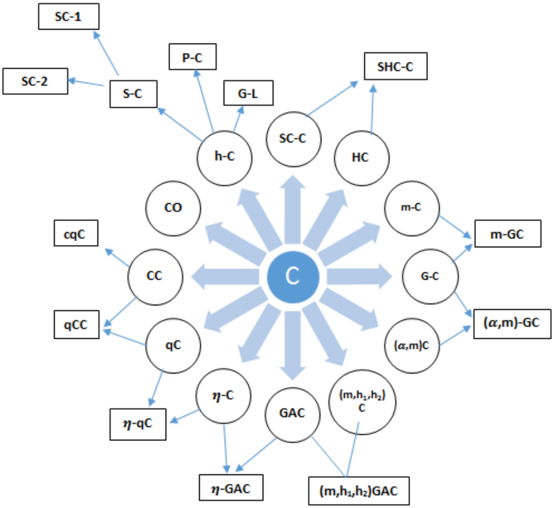


Figure: Diagram

1) **C** Classical convex function.
2) **CO** Convexity with respect to another function.
3) **SC-C** Strongly convex function with modulus C.
4) **HC** Harmonically convex function.
5) **SHC-C** Strongly harmonically convex function with modulus C.
6) **h-C** h-convex function.
7) **P-C** P- convex function.
8) **G-L** Godunova-Levin function.
9) **S-C** s-convex function.
10) **SC-1** s-convex function in the first sense.

11) **SC-2** s-convex function in the second sense.
12) **m-C** m-convex function.
13) **G-C** Geometrically convex function.
14) **m-GC** m-geometrically convex function.
15) **(α,m)-C** (α,m) convex function.
16) **(α,m)-GC** (α,m) geometrically convex function.
17) **GAC** Geometeric arithmetically convex function.
18) **(m,h_1, h_2)-C** (m,h_1, h_2) convex function.
19) **(m,h_1, h_2)-GAC** (m,h_1, h_2) geometric arithmetically convex function.
20) η-**C** η convex function.

21) η-**GAC** Generalized geometric arithmetically convex function with respect to η.
22) **qC** Quasi-convex function.
23) **CC** C- convex function.
24) **cqC** C- quasi-convex function.
25) **qCC** Quasi-convex function with respect to C.
26) η**qC** η-quasi-convex function.

The Hermite-Hadamard Inequality is one of the topics that attracts the most attention in the Mathematical Sciences today (see [1, 52]). This relevance is because this inequality establishes a relationship between the mean of a convex function and its value at the midpoint of an interval.

In the last 25 years, we have witnessed a great growth in the number of researchers and their productions, interested in the Hermite-Hadamard Inequality. These productions have focused on the following work directions:

1) Using different notions of convexity (see [3],[5],[6],[7],[8],[11],[19],[25],[30],[37],[40],[44],[58],[67]).

2) Refinement of the mesh used (there is a crucial issue in this direction of work, suppose we use instead of a and b, the ends of the interval, the points a, $\frac{a+b}{2}$ and b, then we must ensure that at the midpoint, the integral operator used, does not have a jump, since the result would not be guaranteed in all $[a,b]$).

3) Improved estimates of the left and right members of (0.2) (see [12]).

4) Using new generalized and fractional integral operators (see [4],[9],[10],[14],[15],[16],[20],[21],[22],[23],[24],[26],[27],[29],[39],[41],[42],

In the aforementioned works, there are enough references so that the interested reader can form an important database.

The Hermite-Hadamard Inequality

On 22 November 1881 Ch. Hermite (1822-1901) sent a letter to the journal Mathesis. An extract from that letter was published in Mathesis 3 (1883), p. 82. It reads: “Sur deux limites d’une integrale definie. Soit $f(x)$ une fonction qui varie toujours dans le m me sens de $x = a$, Ã $x = b$. On aura les relations

$$(b-a)f\left(\frac{a+b}{2}\right) < \int_a^b f(x)dx < (b-a)\frac{f(a)+f(b)}{2} \tag{0.3}$$

ou bien

$$(b-a)f\left(\frac{a+b}{2}\right) > \int_a^b f(x)dx > (b-a)\frac{f(a)+f(b)}{2} \tag{0.4}$$

suivant que la courbe $y = f(x)$ tourne sa convexité ou sa concavité vers l’axe des abcisses. En faisant dans ces formules $f(x) = 1/(1+x)$, $a = 0$, $b = x$, il vient

Hermite's note is not recorded in the referative journal Jahrbuch fiber die Fortschritte der Mathematik, nor in Hermite's collected papers [62] which were published "sous les auspices de l'Académie des sciences de Paris par Émile Picard, membre de l'Institut".
E. F. Beckenbach [13], p. 441, writes that the first inequality in (0.2) was proved in 1893 by J. Hadamard [31]; see, in particular, pp. 174-176, 186. Beckenbach used great skill in order to recognize that Hadamard obtained the first inequality in (0.2), although it was explicitly published by Hermite ten years earlier. Beckenbach, though undoubtedly an expert in the history and theory of convex functions, was not aware of Hermite's result.

First step

A new way to define an integral operator, and take a first step in generalizing a known result, is to consider a certain weight in the definition of the operator integral, as follows:

Definition

(see [4]) Let $\phi \in L_1[a_1, a_2]$ and let w be a continuous and positive function, $w : I \to \mathbb{R}$, with first derivative integrables on I. Then the weighted fractional integrals are defined by (right and left respectively):

$$I^w_{a_1+}\phi(t) = \int_{a_1}^{t} w'\left(\frac{a_2 - t}{a_2 - a_1}\right)\phi(t)\,dt, \quad t > a_1 \qquad (0.6)$$

$$I^w_{a_2-}\phi(t) = \int_{t}^{a_2} w'\left(\frac{t - a_1}{a_2 - a_1}\right)\phi(t)\,dt, \quad t < a_2. \qquad (0.7)$$

Lemma

([18]) Let $f : I \in \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I, $a, b \in I$ with $a < b$. If $f' \in L[a, b]$, then the following equality holds:

$$\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx=\frac{b-a}{2}\int_0^1 (1-2t)f'(ta+(1-t)b)dt. \tag{0.8}$$

the above could be rewritten as

$$\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx=$$
$$\frac{b-a}{2}\int_0^1 t\left[f'(ta+(1-t)b)-f'((1-t)a+tb)\right]dt. \tag{0.9}$$

First step

Lemma

([1]) Let $f : I \in \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I, $a, b \in I$ with $a < b$. If $f \in L[a,b]$, and w has first derivative integrable on I then the following equality holds:

$$\begin{aligned} & (w(1) - w(0))\,(f(b) - f(a)) - \frac{1}{b-a}\left[I^w_{a+} f(b) + I^w_{b-} f(a)\right] \\ = \; & (b-a)\int_0^1 w(t)\left[f'((1-t)a + tb) - f'(ta + (1-t)b)\right] dt \end{aligned} \tag{0.10}$$

In [4] we presented the following definitions.

Definition

Let $h : [0,1] \to \mathbb{R}$ be a nonnegative function, $h \neq 0$ and $\psi : I = [0,+\infty) \to [0,+\infty)$. If inequality

$$\psi\left(\tau\xi + m(1-\tau)\varsigma\right) \leq h^s(\tau)\psi(\xi) + m(1-h^s(\tau))\psi\left(\varsigma\right) \quad (0.11)$$

is fulfilled for all $\xi, \varsigma \in I$ and $\tau \in [0,1]$, where $m \in [0,1]$, $s \in [-1,1]$. Then a function ψ is called a (h,m)-convex modified of the first type on I.

Definition

Let $h : [0,1] \to \mathbb{R}$ nonnegative functions, $h \neq 0$ and $\psi : I = [0,+\infty) \to [0,+\infty)$. If inequality

$$\psi\left(\tau\xi + m(1-\tau)\varsigma\right) \leq h^s(\tau)\psi(\xi) + m(1-h(\tau))^s\psi\left(\varsigma\right) \tag{0.12}$$

is fulfilled for all $\xi, \varsigma \in I$ and $\tau \in [0,1]$, where $m \in [0,1]$, $s \in [-1,1]$. Then a function ψ is called a (h,m)-convex modified of the second type on I.

The second result of [18] and which will serve as a basis for commenting on a second generalization is the following (see Theorem 2.2):

Theorem

Let $f : I \in \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I, $a, b \in I$ with $a < b$. If $|f'|$ is convex on $[a, b]$, then the following inequality holds:

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{(b-a)\left(|f'(a)| + |f'(b)|\right)}{8}. \tag{0.13}$$

Now, taking into account Lemma 3 we can generalize the previous result in this form:

Theorem

Let $f : I \in \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I, $a, b \in I$ with $a < b$. If $|f'|$ is (h, m)-convex modified of second type on $[a, b]$, then the following inequality holds:

$$
\begin{aligned}
& \left| \left((w(1) - w(0)) \left(f(b) - f(a) \right) - \frac{1}{b-a} \left[I^{w}_{a+} f(b) + I^{w}_{b-} f(a) \right] \right| \\
\leq \; & (b-a) \left\{ \left(\left| f'(a) \right| + \left| f'(b) \right| \right) \int_0^1 w(t) h^s(t) dt \right. \\
+ \; & \left. m \left(\left| f'(a/m) \right| + \left| f'(b/m) \right| \right) \int_0^1 w(t) (1 - h(t))^s dt \right\}. \qquad (0.14)
\end{aligned}
$$

Third step

A last step in the generalization process is to consider the argument of the function, dependent on a parameter, so instead of working with $(1-t)a+tb$ we would work with $\frac{(1-t)a}{n+1}+\frac{(n+t)b}{n+1}$, with n a natural number for example. So, we have the final result, the proof of which is left to the reader.

Lemma

Let $f : I \in \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I, $a, b \in I$ with $a < b$. If $f \in L[a,b]$, and w has first derivative integrable on I then the following equality holds:

$$
\begin{aligned}
& \left[w(1)(f(b)+f(a)) - w(0)\left(f\left(\frac{a+nb}{n+1}\right)+f\left(\frac{na+b}{n+1}\right)\right)\right] \\
- & \left(\frac{n+1}{b-a}\right)\left\{J^{w}_{\frac{a+nb}{n+1}+}f(b) + J^{w}_{\frac{na+b}{n+1}-}f(a)\right\} \\
= & \frac{b-a}{n+1}\int_0^1 w(t)\left[f'\left(\frac{(1-t)a}{n+1}+\frac{(n+t)b}{n+1}\right)\right. \\
- & \left. f'\left(\frac{(n+t)a}{n+1}+\frac{(1-t)b}{n+1}\right)\right] dt.
\end{aligned}
$$

Conclusions

In this talk we have presented some methodological notes, which we have illustrated with a well-known classic result (Lemma 2.1 of the work [18] that has received more than a thousand citations, which clearly speaks of its seminal importance).
We have shown the fundamental steps that allow us to obtain new generalizations of the aforementioned Lemma. First the underlying idea that two functions can be used in said Lemma instead of just one, through a variable change. The introduction of weighted integrals, which allows us to give results for other integral operators, even fractional ones. The third step is the consideration of a new notion of convexity, the modified (h, m)-convex functions of the second type, which encompass many of the known definitions of convexity. And finally, the consideration of a functional argument that depends on such a parameter, instead of giving a new equality, we are giving "families" of equalities, which shows the breadth of the last Lemma presented above.

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