Empirical Evaluation and Diagnostic Robustness of GrünwaldLetnikov Fractional Derivatives in Macroeconomic Modeling:

Evidence from Senegals GDP

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Outline

1. Motivation
2. Methodology
3. Main Results
4. Diagnostics
5. Sensitivity Analysis
6. Conclusion

Motivation

- GDP dynamics often exhibit persistence and memory effects.
- Classical integer-order transformations use fixed accumulation or differencing.
- Fractional calculus allows non-integer differentiation orders.
- This flexibility may better capture heterogeneous macroeconomic dynamics.

Research question

Does a Grünwald–Letnikov fractional regression model improve the in-sample representation of Senegal's GDP compared with a transformed linear model?

Objective

- Model Senegal's GDP using seven macroeconomic indicators.
- Compare two specifications:
  - Transformed linear model.
  - Grünwald–Letnikov fractional regression model.
- Evaluate in-sample fitting accuracy.
- Assess residual diagnostics and sensitivity to fractional-order bounds.

Methodology: Two Models

Transformed linear model

$$GDP_t = \sum_{k=1}^{3} \beta_k x_{k,t} + \beta_4 I(x_4)_t + \beta_5 x_{5,t} + \beta_6 x_{6,t} + \beta_7 \Delta x_{7,t} + \varepsilon_t.$$

Grünwald–Letnikov fractional model

$$y_t = \sum_{k=1}^{7} c_k D_{GL}^{\alpha_k}(x_{k,t}) + \varepsilon_t.$$

- Each explanatory variable has its own fractional order α_k.
- Fractional orders are selected by minimizing in-sample MSE.

Evaluation Criteria

- Mean absolute deviation: MAD.
- Mean squared error: MSE.
- Coefficient of determination: R^2.
- Corrected Akaike information criterion: AICc.
- Bayesian information criterion: BIC.
- Residual diagnostics: Jarque–Bera, Breusch–Pagan, Breusch–Godfrey, and Durbin–Watson.

Estimated Fractional Orders

Variable	$\widehat{\alpha}_k$
x_1	0.153340
x_2	-0.050623
x_3	0.092054
x_4	0.399062
x_5	-0.069613
x_6	0.195035
x_7	-0.207090

- Fractional orders differ across variables.
- The largest positive order is for x_4:

$$\widehat{\alpha}_4 = 0.399062.$$

Main Result: Model Comparison

Model	MSE	MAD	AICc	BIC
Transformed linear	8.64991×10^{16}	239,345,074	1357.105	1363.482
GL fractional	2.436593×10^{16}	122,474,039	1332.992	1332.256

Key finding

The GL fractional model reduces the in-sample MSE by approximately 71.8% and yields lower MAD, AICc, and BIC values than the transformed linear model.

Interpretation

- The GL model exhibits lower in-sample fitting errors.
- Lower AICc and BIC values indicate lower information-criterion values after accounting for model complexity.
- The transformed linear model has a slightly higher R^2, whereas the GL model has lower MSE, MAD, AICc, and BIC.

Residual Diagnostics

Test	Statistic	p-value	Interpretation
Jarque–Bera	0.97057	0.6155	Normality not rejected
Breusch–Pagan	6.2175	0.3993	No heteroskedasticity
Breusch–Godfrey	3.2508	0.07139	Not significant at 5%
Durbin–Watson	1.3884	0.006744	Positive autocorrelation

Important finding

The Durbin–Watson statistic indicates positive first-order residual autocorrelation in the GL model.

Sensitivity Analysis

Bounds	MSE	MAD	R^2
$[-1, 1]$	2.436593×10^{16}	122,474,039	0.9992014
$[-1.5, 1.5]$	2.436593×10^{16}	122,476,260	0.9992014
$[-2, 2]$	2.436593×10^{16}	122,476,158	0.9992014

Result

Model performance remains stable when the bounds on the fractional orders are widened from $[-1, 1]$ to $[-2, 2]$.

Conclusion

- The GL fractional model exhibits lower MSE, MAD, AICc, and BIC values.
- It reduces the in-sample MSE by approximately 71.8%.
- The results are stable to alternative bounds on the fractional orders.
- Residual diagnostics reveal positive autocorrelation, which should be addressed in future extensions.

Main message

Grünwald–Letnikov fractional transformations provide a flexible in-sample representation of Senegal's GDP dynamics.

Future Work

- Correct for residual serial correlation.
- Use autoregressive error structures or dynamic specifications.

Thank you