REGULARIZED RECONSTRUCTION OF SCALAR PARAMETERS IN SUBDIFFUSION WITH MEMORY

NATALIYA VASYLYEVA

Fractional differential equations are an effective mathematical tool to study anomalous diffusion phenomena, which are structurally presented in real-life models such as oil pollution, tumor growth, oxygen transport through capillaries to tissues, heat conduction in living tissues. The feature of these anomalies in diffusion/transport processes is that the mean square displacement of the diffusing species $\langle(\Delta x)^2\rangle$ scales as a nonlinear power law in time, i.e. $\langle(\Delta x)^2\rangle \sim t^\nu$, $\nu > 0$. For a subdiffusive process, the value of ν belongs to $(0,1)$, while for normal diffusion $\nu = 1$, and for superdiffusive process, we have $\nu > 1$. Appealing to the fractional differential equations, this subdiffusion order ν appears as the order of the leading fractional derivative in time, $\mathbf{D}_t^\nu$, in the corresponding one- or multi-term fractional differential operator $\mathbf{D}_t$ with (in general) variable coefficients. A particular case of this operator reads

$$\mathbf{D}_t = \rho(x,t)\mathbf{D}_t^\nu + \sum_{i=1}^{M} \rho_i(x,t)\mathbf{D}_t^{\nu_i}, \quad \rho(x,t) > 0, \quad 0 \leq \nu_1 < \nu_2 < ... < \nu_M < \nu < 1.$$

However, sometimes a value of the subdiffusion order (the order of the fractional derivative in time) is not given a priori. In this talk, we discuss a very effective (in practice) technique to reconstruct the orders of fractional derivatives, coefficients in the fractional operator $\mathbf{D}_t$ and the order $\gamma \in (0,1)$ of a singularity in the memory kernel $\mathcal{K} = t^{-\gamma}\mathcal{K}_0(t)$ in a semilinear subdiffusion equation

$$\mathbf{D}_t u - \mathcal{L}_1 u - \int_0^t \mathcal{K}(t-\tau)\mathcal{L}_2 u(x,\tau)d\tau = g(x,t,u),$$

where $\mathcal{K}_0$ and g are given functions, $\mathcal{L}_1$ and $\mathcal{L}_2$ are uniform elliptic operators of the second order with time and space depending coefficients. It is worth noting that, the particular case of this equation models the oxygen transport through capillaries to tissue.

In order to recognize these scalar parameters, we analyze the corresponding inverse problem with additional local or nonlocal measurement $\psi(t)$ for small time interval, $t \in [0, t^*]$, $t^* << 1$. In fine, collecting Tikhonov regularization scheme and quasioptimality approach, we describe a computational algorithm to reconstruct these parameters in the case of discrete noisy observation $\psi_\delta(t_i)$, $i = 1, 2, ..., K$.

Institute of Applied Mathematics and Mechanics of NAS of Ukraine
G.Batyuka st. 19, 84100 Sloviansk, Ukraine; and
S.P. Timoshenko Institute of Mechanics of NAS of Ukraine
Nesterov str. 3, 03057 Kyiv, Ukraine
E-mail address, N.Vasylyeva: nataliy_v@yahoo.com