Existence and Decay Rate of Global Solutions for a Second-Order Evolution Equation with Memory and Nonlinear Time-Varying Delay

Abidi Khedidja*

International Conference on Applied Mathematics and Mathematical Modelling (ICAMMM 2025)

Abstract

This work investigates the existence and decay properties of global solutions for a class of second-order evolution equations that incorporate memory effects, a nonlinear time-varying delay term, and a decaying weight function. Such models arise in viscoelastic and thermoelastic media where the material response depends on past deformations and delayed feedback mechanisms. Under suitable conditions on the memory kernel, delay function, and nonlinearity, we establish the global existence of weak solutions using Galerkin approximations and compactness methods. Furthermore, we derive general decay estimates for the associated energy functional by constructing an appropriate Lyapunov functional. The results extend existing work on wave equations with memory by accounting for time-dependent delay effects and variable damping.

Keywords: Evolution equations, Memory, Nonlinear delay, Lyapunov functional, Global solution, Energy decay

Mathematical Model and Hypotheses

We study the following problem:

$$u_{tt}(x,t) - \Delta u(x,t) + \int_0^t g(t-s)\Delta u(x,s)\,ds + \mu(t) f(u_t(x,t-\tau(t))) = 0, \quad x \in \Omega,\ t > 0,$$

with:

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega,$$
$$u(x,t) = 0, \quad x \in \partial\Omega,\ t > 0.$$

Hypotheses:

(H2) The memory kernel $g : \mathbb{R}^+ \to \mathbb{R}^+$ is decreasing, continuously differentiable, and satisfies:

$$g(t) \geq 0, \quad g'(t) \leq 0, \quad \int_0^\infty g(s)\,ds < \infty.$$

*University of Laghouat, Algeria. Email: k.abidi@lagh-univ.dz

(H3) The time-varying delay function $\tau(t)$ satisfies:

$$0 < \tau_0 \leq \tau(t) \leq \tau_1 < \infty, \quad \forall t \geq 0.$$

(H4) The weight function $\mu(t)$ is positive and decreasing:

$$\mu(t) > 0, \quad \mu'(t) \leq 0, \quad \forall t \geq 0.$$

(H5) The nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is locally Lipschitz:

$$|f(v_1) - f(v_2)| \leq L|v_1 - v_2|, \quad \forall v_1, v_2 \in \mathbb{R}, \text{ for some } L > 0.$$